AMYGDALA

$$z \mapsto z^2 + c$$

A Newsletter of $\mathcal{M}$ -- the Mandelbrot Set
AMYGDALA, Box 219, San Cristobal, NM 87564

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THE SLIDE

As was the case in issue #2, the slide enclosed with this issue was created by John Dewey Jones, who wrote:

"Slide 35 is of the quasi-Mandelbrot set obtained from

$$z_n = z_{n-1}^{2.5}+c$$

> The viewing area is the 3x2 box centered on the origin. The slide shows the strange results typical of non-integer values for the exponent: fractal curves very similar to those for the nearest integral k, but with lines of discontinuity running though them, like shock waves. These discontinuities are also reproduced on smaller scales."

These "fracture lines" are probably the result of the multivaluedness of $z^{2.5}$. For integer exponents, z^n has a unique value for each z, but not so for non-integer exponents — for example $(-1)^{2.5} = (-1)^2(-1)^{.5} = \pm i$, so we have two "branches" for the function. Unless the computer program deals carefully with multivaluedness, we may well find adjacent sets of pixels whose values are computed using different branches of the function, resulting in these "fracture lines" of discontinuity between the sets.

This is discussed in more detail under **BIFURCATIONS.**

MORE SLIDES!

As of today, 7 subscribers have given us "a sincere vote in favor of getting 25 slides (say) over the same period as issues 1-10", so our second-tier subscription for a color-slide supplement is a reality! I haven't gotten it together to produce these slides as yet, so I can't say when they will start to appear — but they will definitely appear as promised!

BIFURCATIONS

Let's look in greater detail at the iterations involved in process underlying the slide: $z \mapsto z^{2.5} + c$.

Let $z = x+iy = r(\cos\theta + i \sin\theta)$. Then $z^{2.5}$ has two values:

$r^{2.5}(\cos(2.5\cdot\theta) + i\sin(2.5\cdot\theta))$ and

$r^{2.5}(\cos(2.5\cdot\theta+\pi) + i\sin(2.5\cdot\theta+\pi))$

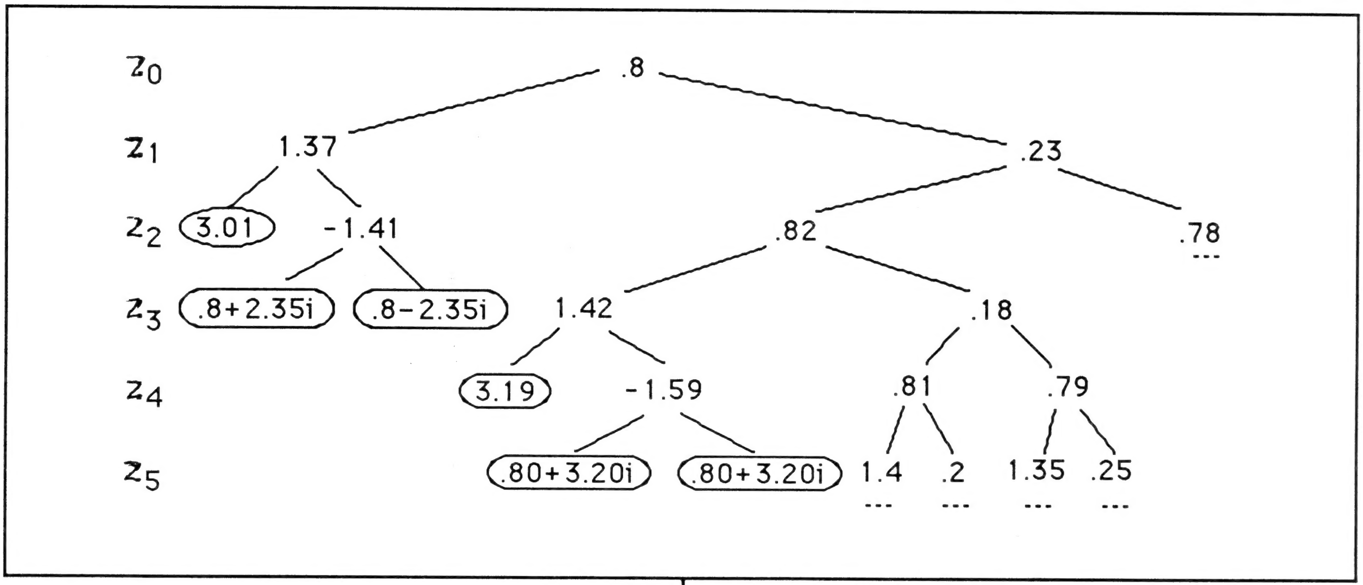
Iterating each of these in turn produces two values, and so on, resulting in a tree of bifurcations.

Let's examine this tree of bifurcations for $c = 0.8$, terminating each branch if $|z_n| > 2$ (see the figure on page 2).

The starting point, $z_0 = c = 0.8$, is shown at the top. One iteration produces two descendants: $z_1 = 1.37$ and $z_1 = 0.23$. Of the descendants of 1.37, $z_2 = 3.01$ and $z_2 = -1.41$; the former is circled, indicating that its modulus exceeds 2. In the ordinary course of events we would conclude that $c = 0.8$ has dwell 2 — but what of the other branches? As you can see, the possible descendants proliferate, and there is no consistent pattern of dwell. Some of the descendants, shown with an ellipsis below them, continue on for more than five iterations.

If the dwell algorithm does not take this branching pattern of iteration into account, it will likely choose some terminal node haphazardly, and deduce the dwell of the root point from it.

There are ways of dealing with the branching consistently, e.g. exploring the tree in width-first fashion, stopping at the first descendant with modulus > 2, and taking its depth as the dwell of the root. This will work for exponents like 2.5,



but for others, like $\sqrt{10}$, for example, the number of branches is infinite!

Oy! Have we got a problem!

M-FICTION

For better or worse John Dewey Jones seems to have created the genre of **M-fiction** with his story, **The Amygdalan Sects**. Here is a second contribution to the genre, **The First Encounter**, by Ronald Lusk.

THE FIRST ENCOUNTER

From the Notebooks of Charles H. Festus, Founder and President of Fractal Growth, Unltd.

Of course, we never expected to break through into the Object. I had always thought of the Object as a computerized curiosity and an outlet for religious crackpots.

The purpose of the test we had set up back then was to continue the search for fundamental particles; somebody had to keep looking. That was the reason I had started "Atom Smasher" in the first place, so that the few hobbyists remaining could continue to exchange information and ideas on the dying science of high energy particle physics.

It was almost eight years from the first issue of "Atom Smasher" until we actually encountered the Object. I can admit now that my motives for starting the newsletter were purely selfish. At the time, I had an old Eurotron 1600 Quark Accelerator that was so clogged with pion dust that it could barely put out 800 TeV (trillion electron volts). The darn thing was the size of a sofa and impossible to find replacement parts for, so I figured that by doing some fancy networking I'd find the parts I needed and meet some interesting people besides.

In a very short time, my "magazine for the particle-collider hobbyist" had caught on and I was being bombarded by readers wanting more and more information. Those requests drove me on, motivating me to the point of achievement until the Object came within my grasp — although I didn't know it. All this was possible because through "Atom Smasher" I met some extremely capable people:— scientists, programmers, engineers — whose skills and knowledge provided endless fascination to me and my readers.

Many of these experts were the relatives of people who had been involved with the notorious Infinitron Project. Susan Wester, our Quantum Programming editor, was the granddaughter of Sir John Henry Wester, originator of the idea of building a gigantic cyclotron about the Earth's equator. Paul Tse's background in micromagnetic accelerators came, of course, from his father Louis Tse — the man who single-handedly turned the Infinitron into a 26,000 mile white elephant. I believe that the warm rivalry between Paul and Susan, stemming from their relatives' historic conflict, helped to give our readers a feeling that we really cared about our forgotten science, regardless of the lack of money or prestige it brought us.

Until we stumbled upon the Object.

The person who was undoubtedly most responsible for our finding the Object was Carl Spring, a man who until he died never forgave himself or us for allowing the Object to be explored. As an old man with only a few years left, I can now say that although I could never rationally agree with Carl's fanatical obsession to keep the Object "unviolated", I have always somehow intuitively understood it.

The media made much of our courtroom battles. Carl was as entitled as were the others or myself to the multiplying fortune that came with opening up the Object to the world, only he spent the bulk of his share in suits or countersuits against either Fractal Growth Unltd. or us personally. It was a very difficult time for all of us, particularly with Carl's unceasing attacks pounding at our hearts and doors. Susan suffered her breakdowns then, and I became a borderline alcoholic, trying to escape simultaneously from the notoriety of finding the Object and from Carl's intrusive moralistic fanaticism. Only Paul stood firm, with his unshakeable faith in science and rationalism providing the lifeline Susan and I clung to.

In the end we all agreed that Carl's vision was wrong, and that the Object existed for all to experience. But that didn't cure the bad taste left in our mouths: we owed Carl. He was the whiz kid who had accelerated Pi mesons before he was twelve. He was the critic and agitator who built up our readership fivefold through his charisma. And he was the innovator who showed us how to amplify our effective power output 1000 times by harnessing the energies of microscopic gravitational singularities. For years we all had trusted and believed in Carl, and finally breaking with him completely was like losing a member of our family. Even now, though, the foundations and organizations he built up around himself continue to hound our every step.

However, on the day of the test we had all been just good friends pursuing a common hobby. Admittedly, this test event was a bit special: it was our first full scale attempt at two million Tev. We'd been priming our readers for it for over a year, hailing the Tse/Spring Tiered Lepton/Skrark Cyclotron as the personal accelerator of the future, capable of achieving undreamed-of energies. And privately, we were pretty hopeful for the project. Susan's running joke was that we were trying to force back to life a science which had self-consciously committed suicide rather than continue to slide endlessly into the abyss of fundamental particles.

As true as that may have been — and we all felt that it was vital that the search be continued regardless of the lack of interest — we never stopped to think that a Shelleyan mutation, a Frankenstein's monster science would arise from our efforts. But in Carl's eyes that was exactly what happened.

For the Object itself, its first capture was a bit of promotional good luck. We had tried to interest the journalistic community in our project, but the only reporter to show was a bored fellow from *New Science* — and his sole reason for even coming was that a nearby international seminar on the asexual behavior of *E. Coli* was cancelled due to disruptions orchestrated by the Moral Majority. The reporter's name was Sind Balland, and he couldn't help but act as if we all owed him a big favor for showing up — so when he decided to observe the test inches from the collision detection chamber, all we could do was hope he'd get a jolt from the experience — which he did.

The goal of the experiment was to collide *skrarks,* the constituents of all leptons, in order to investigate the then frontier of particle physics. As the name implied, our newfangled machine was actually two separate devices. Just as the fusion bombs of the last century depended on a less powerful fission explosion for ignition, so our second, more powerful, stage of acceleration was dependent on a more conventional accelerator as a primer.

The procedure was for tau mesons, the heaviest of the leptons, to be initially accelerated in a modified Tse Microcyclotron. The ensuing collisions would create showers of skrarks, which were then to be fed into Carl's singularity-field cyclotron to be accelerated to the tremendous velocities Carl hoped would be sufficient to allow collisions to occur. Remember that at the time all this was still experimental — especially Susan's operational software. We could run simulations through her mainframe until we were blue, or test the hardware with as many low-powered runs as we liked, but until we actually put down the throttle to two million TeV, there was no way of knowing if we'd get results, or if we'd be able to detect them.

Quite frankly, we were tired of waiting. The capper was Sind shuffling and squawking as we made our final adjustments. I would have thrown him out if I hadn't been so busy helping Carl make singularity calculations.

We hit the switch at 11:56 AM on July 16, 2052; tau mesons were accelerated and collided, the resulting skrarks were accelerated, and at 11:58 our first 2,000,000 TeV collision occurred — and Sind vanished.

I was the only one to witness the disappearance; the others had all been watching their instruments. There was no spark, or flash, or anything spectacular; Sind simply disappeared. Paul was closest to me at his control panel, and I remember grabbing at his shoulder in a panic, not knowing what else to do. At first, Paul couldn't figure out what I was talking about, but then he noticed that Sind was missing and he clued in. He immediately shut down the test. Carl and Susan looked up angrily from their instruments just in time to see Sind reappear just above the spot where he had disappeared, and fall to the floor.

Sind looked as though the Object had taken him by the collar and roughed him up a bit. After a few moments, though, he was coherent enough to tell us about what was the first encounter with the actual, total existence of the Object. Sind himself guessed the true nature of the experience he had just had — that he had travelled within the multi-dimensional space-time of the M object whose existence had been suggested almost a hundred years earlier.

In a very short time we were all making excursions into the Object, flying through its eddies and currents in a full-blown technicolor spectacular. Even Carl joined in the early trips. It wasn't until later, after Sind's reporting of our discovery turned travelling within the endless dilations of the Absolute into a planetary craze, that Carl adopted his negative stance — but by then it was too late. Fractal Growth Unltd. was founded and we were quickly trapped in a future of our own making, entwined forever with the unfolding of time within the Object.

Ronald Lusk
360 E. 55th Street
New York, NY 10022

QUATERNIONS, HO!

A number of readers have written to ask about paradigms similar to the iterative one used to produce two dimensional images of Julia sets and the Mandelbrot set, but which would evoke three dimensional images.

I asked myself the same question a while ago, and put on my "stack" the idea of investigating **quaternions**. The quaternion number system was invented by Sir William Rowan Hamilton (1805-1865), the Irish mathematician and astronomer, who is better known for **Hamilton's principle** (the principle of least action).

Quaternions, which can be thought of as quadruples of real numbers — in much the same way that complex numbers can be thought of as pairs of real numbers — form a number system in which addition and multiplication are defined, the latter being non-commutative: ab is not necessarily equal to ba. Iterations like $q \mapsto q^2+c$ are meaningful for quaternion q, and their properties might well yield interesting four-dimensional pictures analogous to the more familiar two-dimensional pictures associated with $z \mapsto z^2+c$. I don't know how to draw in four dimensions, but it would be interesting to have a look at two- or three-dimensional projections of these four-dimensional beasts.

Serendipitously, there arrived yesterday in the mail a Care package from Clifford Pickover at IBM, containing a copy of the first issue of **The Journal of Chaos and Graphics** (see Bibliography, #40, this issue). The contents include **Quaternions and their Graphics**, by Alan Norton. I'm reproducing the article below since I feel it will be of interest to many Amygdala readers. Two figures accompany the article, which unfortunately I cannot reproduce here. The first looks like a cross-section of a heart, showing the striae of cardiac muscle; the second looks like taffy wound onto a camshaft.

Quaternions and their Graphics

by Alan Norton

Alan Norton is manager of the Parallel Architecture and Applications group at the IBM Watson Research Center, Yorktown Heights, NY 10598.

Recent mathematical work on the dynamics of complex analytical functions has given rise to a new subject matter for computer graphics. The combination of mathematical theory and computer graphics has resulted in new insight into the nature of some of the simplest of mathematical objects, second degree polynomials. Most of that work has focused on the possibilities within the two-dimensional complex plane. I have focussed on how these investigations may be extended to higher dimensions, resulting in shapes that naturally reside in 4-dimensional "quaternions". Note that though they may appear to be "chaotic looking", their behavior and structure are very deterministic. Quaternions may be considered as a way of extending the notion of "number" to four dimensions: The rules of quaternion multiplication and addition provide a way of doing arithmetic on four-dimensional quantities. Quaternions can be represented in

$$Q = a_0 + a_1 i + a_2 j + a_3 k \qquad [1]$$

where i,j, and k are (like the imaginary number i) unit vectors in three orthogonal directions, perpendicular to the real axis. To add or multiply two quaternions, we treat them as polynomials in i, j, and k, but use the following rules to deal with products:

$$i^2 = j^2 = k^2 = -1 \qquad [2]$$

$$ij = -ji = k, \quad jk = -kj = i, \quad ki = -ik = j \qquad [3]$$

The following figures [not shown here in **Amygdala**] represent 3-D "slices" produced by iterating quaternion polynomials (see (1) for more information).

For Further Reading

1. Norton, A. (1982) "Generation and display of geometric fractals in 3-D", Computer Graphics (ACM-SIGGRAPH), Vol 16.

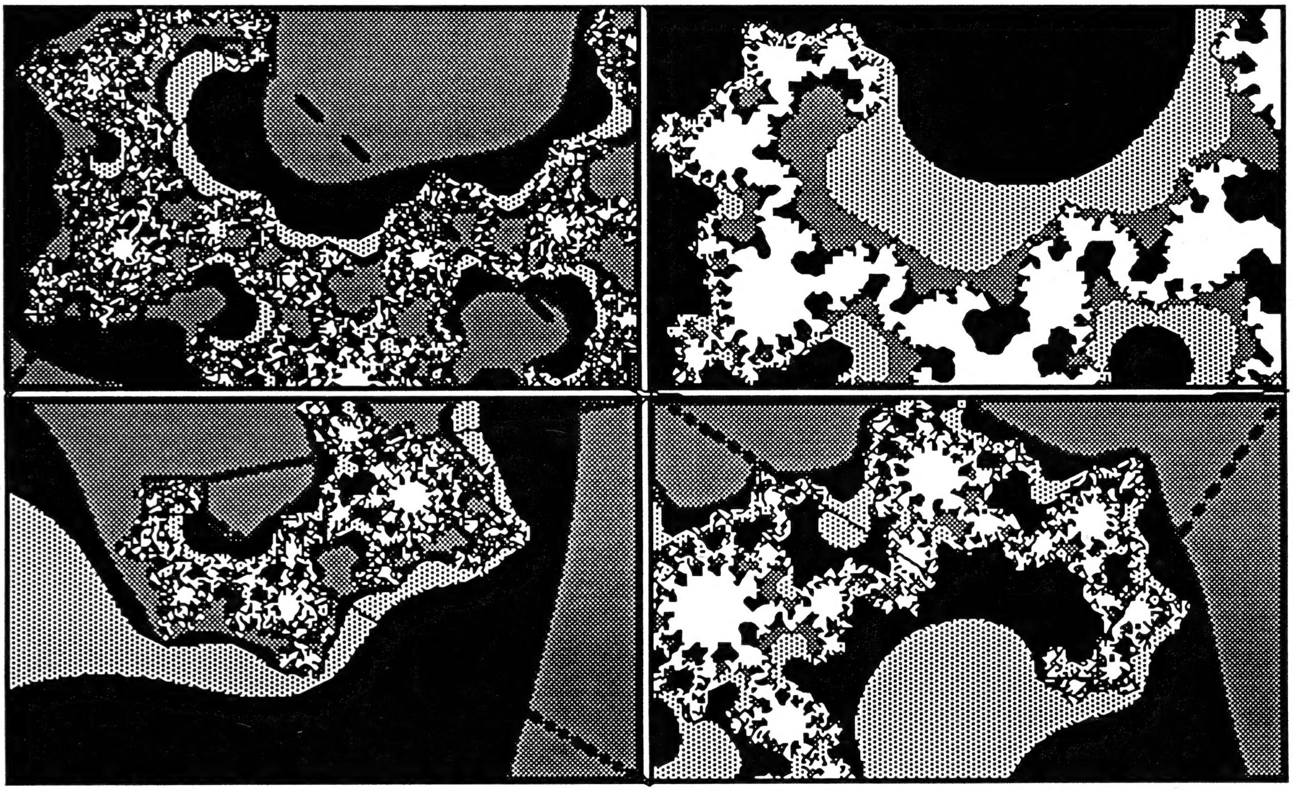
FRACTAL SOFTWARE REVIEWS

For the Macintosh

FRACTALS, by T.D. Scholz, M.D. (Four Snapshots of images included). Dr. Scholz writes:

> The enclosed program runs on the Macintosh and is written in Lightspeed C. No "shortcuts" are used in the iterative process and all variables are double precision with 80 bits of significance. The maximum dwell number is 80.
>
> Upon starting **Fractal**, the screen is divided into four regions and the entire Mandelbrot set is drawn into the upper left region. When it's done drawing, or at any time during the calculation, the user can select a portion of the set to be magnified by a factor of 10. This process can be repeated on the original or magnified images. The magnified areas are placed in successive screen regions in a counterclockwise fashion on the screen. This counterclockwise sequence can be altered by choosing a different region to "draw to..." in the "Draw" menu. A trivial choice in the "View" menu is the "Translate" option which results in no magnification of the set, simply a frame shift down and to the right. The user selects the new upper left corner of the subsequent view.
>
> Images can be printed or stored on the disk in MacPaint format with Command-Shift-3 (some "snapshots" are included on the disk). While dividing the screen into quarters results in some loss of detail, their shorter calculation time allows for more areas to be explored.
>
> If the actual code of this program is of interest to anyone, let me know and I will send it along.

"Image 2.2" from the **Fractals** disk is reproduced at the top of the next page, full-scale.



A copy of this diskette (double-sided, 3.5" Sony, recorded single-sided) may be had from **Amygdala** for $7.50, postpaid.

KaleidoScope, by Mark W. Bolme.
Available from: Sintar Software; PO Box 3746; Bellevue, WA 98009 for $25.00.
About Sintar, Mark Bolme writes:

> Sintar Software is a small company specializing in software to enable exploration of concepts that are both entertaining and intellectually stimulating. Examples of this type of concept are Mandelbrot set generation, the "game" of life, 3-D life, and neurode-controlled synthetic "creatures".
>
> We have been developing primarily for the IBM™ PC. I have tried to convince my partner that we should expand to the Macintosh™ market, because I believe that the Macintosh market is more inclined to enjoy this type of software. He is worried about the size of the market, but I convinced him to try two programs (KaleidoScope and FractalMagic).

About KaleidoScope itself, Mark writes:

> This program starts at a point, and then uses an equation to generate the next point (which is plotted on the screen). As this is repeated for point after point "one has the impression miniature electronic rain in falling on the screen" resulting in "stunning, highly detailed images" (A. K. Dewdney, "Computer Recreations", September, 1986 *Scientific American*). In theory, you can continue this calculation forever.

The iteration begins with a complex number $z = x+iy$, and real constants A, B, and C. The calculation proceeds by iterating:

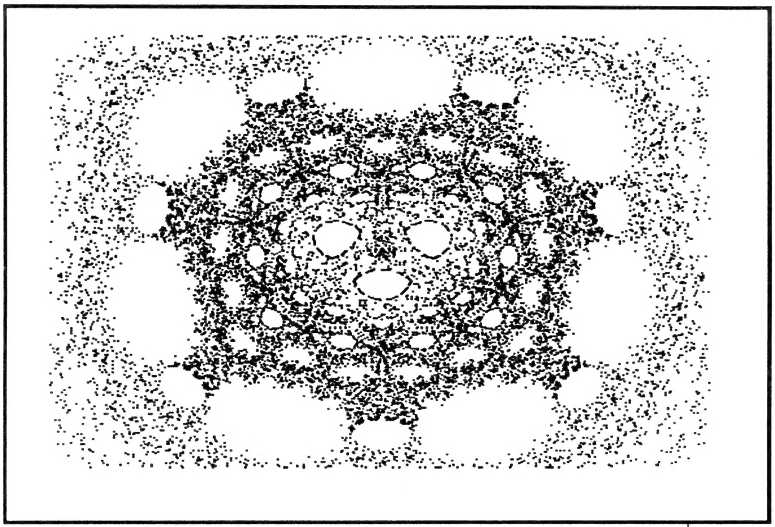
$$x+iy \mapsto (y - \mathrm{sign}(x)\sqrt{|Bx-C|}) + i(A-x)$$

plotting (x,y) after each iteration.

The function sign(x) is equal to 1 if x is positive, and -1 if x is negative.

KaleidoScope allows you to: change the parameters A, B, and C, rotate the image to view it from any angle, change the initial seed (starting x and y), change the area of the X-Y plane that is plotted, auto scale, select a sub-region, and reverse the presentation to white points on a black background.

The figure at the top of the next page shows the result of starting with $x = y = 0$, $A = 12$, $B = 10$, $C = 52$, (taken from the author's birthday!) and iterating for about 20 minutes.



BIBLIOGRAPHY

Items #39,40,43 were submitted by Clifford A. Pickover. Items #41 and #42 appear in #40.

37. JP Crutchfield, JD Farmer, NH Packard. "Chaos". Scientific American (December 1986) 46-57.

38. LM Sander. "Fractal Growth". Scientific American (January 1987) 94-100.

39. Clifford A Pickover & Elahe Khorasani. "Computer Graphics Generated from the Iteration of Algebraic Transformations in the Complex Plane". Comput. & Graphics (9,2) 1985. pp 147-151.

40. Clifford A Pickover (Ed). The Journal of Chaos and Graphics, Volume 1, April 1987. ["This journal publishes articles, notes, reviews, notices, computational recipes, and correspondence on the subject of the graphics of chaos. JCG will involve the study of how complicated behavior and structures can arise in systems which are based on simple rules. Topics for the journal include: iteration, cellular automata, bifurcation maps, Julia sets, Life, creation of the patterns of nature from simple rules, phase plane portraits..., primarily aesthetic and unusual graphics derived from mathematics." Available as RA 186 from IBMDS (IBM Distribution Services; see * footnote). Two individual articles are cited below as #41 & #42]

* Copies may be requested from:
IBM Thomas J. Watson Research Center
Distribution Services 73-F11
PO Box 218
Yorktown Heights, NY 10598
but you must give them a document number!

41. Alan Norton. "Quaternions and their Graphics". The Journal of Chaos and Graphics, 1 (April 1987). 3-4. [Reproduced (except for 2 figures) in this issue of Amygdala #3, page 4]

42. Kevin McCarty. "Piecewise Linear Recurrences". The Journal of Chaos and Graphics, 1 (April 1987). pp 12-16.["In this article I discuss iterated mappings on the plane for a family of piecewise linear formulas suggested by the Computer Recreations column in the September 1986 issue of Scientific American. Examples are presented of the complex behavior these simple formulas can generate, along with techniques for investigating these phenomena on a computer. Some mathematical analysis is adduced to explain some of the observed features". From the Introduction]

43. CA Pickover. "Biomorphs: Computer Displays of Biological Forms Generated from Mathematical Feedback Loops". Computer Graphics Forum 5 (1986) 313-316. ["Abstract: A computer graphics algorithm is used to create complicated forms resembling invertebrate organisms. These natural morphologies are generated through the iteration of mathematical transformations. Several illustrations are chosen as examples of the diversity of biological structures which result from this technique." The illustrations have to be seen to be believed!]

MAIL-ORDER BOOKS ABOUT FRACTALS: **THE BEAUTY OF FRACTALS** lists for $35.00, and **THE FRACTAL GEOMETRY OF NATURE** lists for $34.95. Amygdala subscribers can get either one from us for $31.50, postpaid. Please allow 4-6 weeks for delivery, ARO.

COMMERCIAL PRODUCTS

ART MATRIX; PO Box 880; Ithaca, NY 14851 USA. (607) 277-0959. Prints, FORTRAN program listings, 36 postcards $7.00, sets of 2 packs $10.00, 140 slides $20.00. Or send for FREE information pack with sample postcard. Custom programming and photography by request. Make a bid.

GRAPHICUS DESIGN CO; 24 Ridgeley Ave; Fairfield CT 06430; (203) 368-3448. Fractal T-shirts, Black or beige with a gold-to-red gradation of color: "Official Fractal Technician" or "Member: Mandelbrot Set $Z_1 = Z_0^2 + C$"; $9 postpaid. Write for brochure of other Fractal novelties — caps, stickers, etc.

CIRCULATION

As of today (April 14, 1987) Amygdala has 131 paid-up subscribers and the newsletter will be sent gratis to 10 others — until they pay up!